USING OPTIMALLY CONNECTED PAIRS TO APPROXIMATE TSP SOLUTIONS

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TRAVELLING SALESMAN PROBLEM

"Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?“

- Complete Undirected Weighted Graph
- Brute Force Required To Guarantee Optimal Solution
- (n-1)! Calculations

NON GUARANTEED SOLUTIONS

- Do Not Guarantee Optimal Solution
- May Be More Efficient
- In Most Cases, Good Enough

WEIGHTED COMPLETE UNDIRECTED GRAPHS

- **Each edge is assigned a “distance” or “cost” value**
- **Each node is connected to all other nodes by an edge**
- **It costs the same to traverse an edge in one direction as the other**

SMALLEST EDGE OF A NODE

Consider a Weighted Complete Undirected graph (WCU graph). If node k is in the WCU graph, of all the nodes connected to node k, the node min(k) is the closest to k; that is, the edge(k, min(k)) has the smallest weight of all the edges connected to node k. Every node within a WCU graph has one or more min(node).

Therefore, we define *min(k)* as the closest node to k.

OPTIMALLY CONNECTED PAIRS

In the case of a WCU graph, if there exists two nodes, a and b, where:

min(a)=b and min(b)=a

Then we define these nodes as an *Optimally Connected Pair* (OCP). An OCP contains two nodes, of which are closer to each other than any other nodes within the graph.

THEOREM

For every WCU graph, there exists <u>at least one</u> OCP comprised of nodes within the WCU graph.

PROOF

- **Proved using Proof By Contradiction**

Consider a WCU graph G such that:

a. $|G| = n$

b. $G = \{g_1, g_2, g_3, \ldots, g_m, g_{m+1}, \ldots, g_{n-1}, g_n\}$

c. G has no OCP's.

If there are no OCP's in WCU graph G, then:

$\min(g_1) = g_2$ $\min(g_2) = g_3$ … $\min(g_{n-1}) = g_n$ $\min(g_n) = g_m$

However, if the aforementioned is true, then:

$|(g_1, g_2)| > |(g_2, g_3)| > \ldots > |(g_{n-1}, g_n)| > |(g_n, g_m)| > |(g_m, g_{m+1})|$

Simplified,

$$|(g_m, g_{m+1})| > |(g_n, g_m)| > |(g_m, g_{m+1})|$$

(Not Possible)

So, a WCU cannot exist without an OCP.

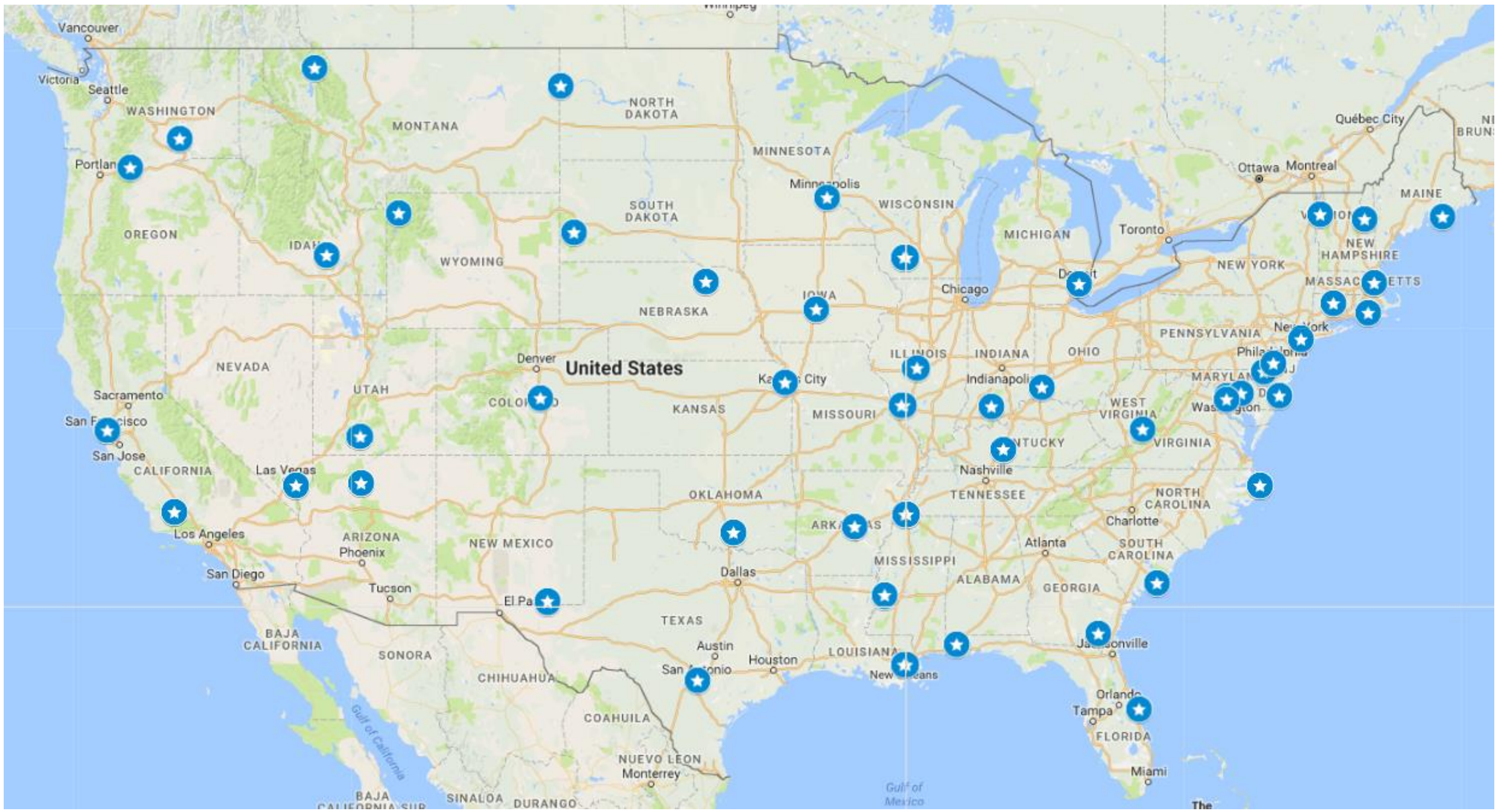
OCP ALGORITHMS

- **Take into account OCP's to efficiently find paths in TSP Problems**
- **Similar to Greedy Algorithms, but much more accurate**
- **Extremely fast and cost efficient**
- **Worst Case: n^3 calculations**
- **Best Case: n calculations**
- **Average: $0.5n^3 + 0.5n$ calculations**
- **M^2 memory required**

TOUR OF 50 USA LANDMARKS

- Originally covered by Newsweek in 1954
- Challenge by Rand Corporation
- 50 Nodes, 6.08×10^{62} calculations to Brute Force
- Believed to be Solved

DATA SET



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Victoria
Seattle
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Portland
OREGON
MONTANA
IDAHO
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NORTH DAKOTA
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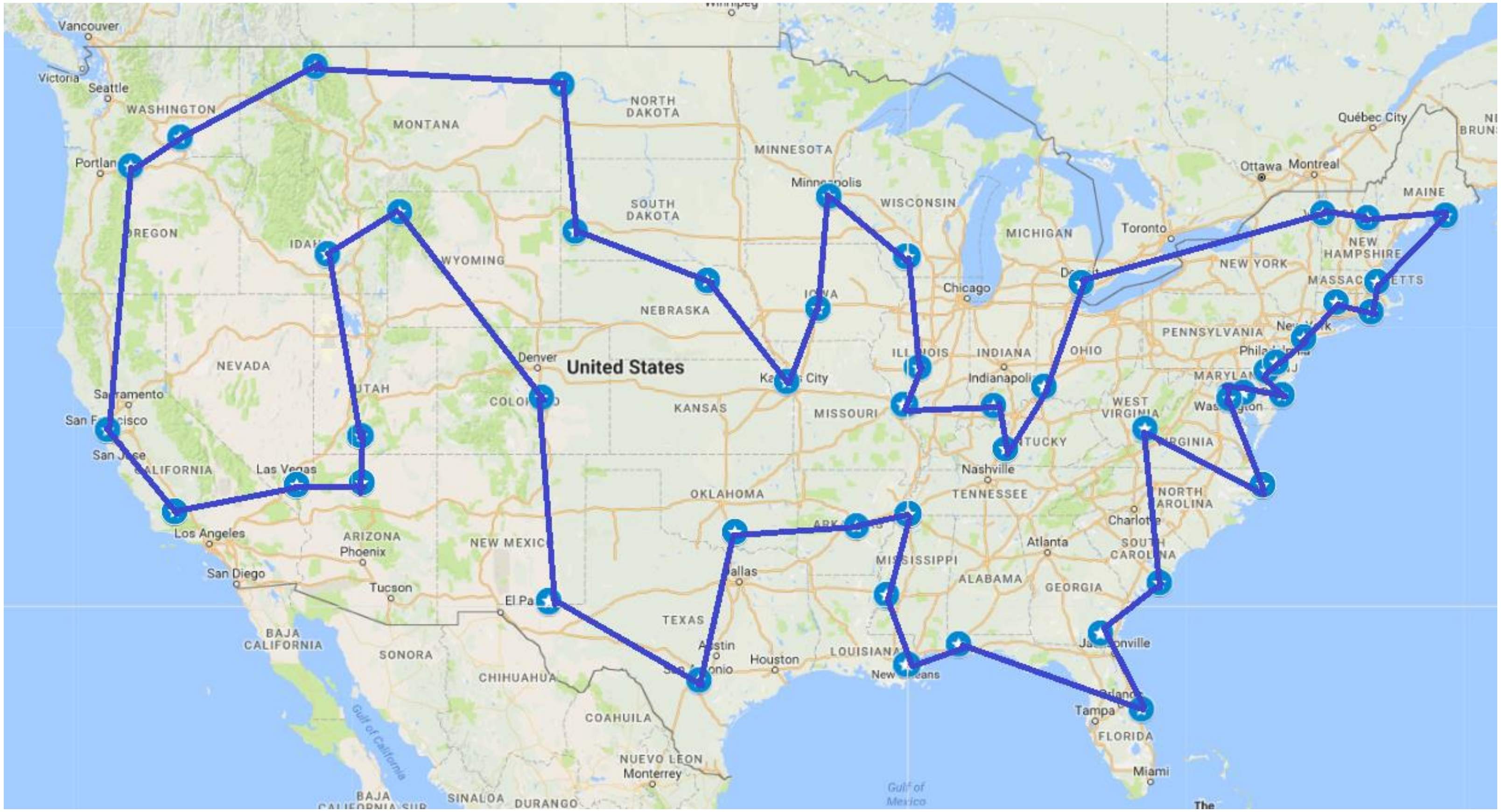
NEBRASKA
MINNESOTA
WISCONSIN
MICHIGAN
Chicago
Toronto
Ottawa
Montreal
Québec City
MAINE
NEW HAMPSHIRE
NEW YORK
PENNSYLVANIA
OHIO
INDIANA
Indianapolis
ILLINOIS
IOWA
MISSOURI
KANSAS
Denver
United States
NEVADA
UTAH
Sacramento
San Jose
CALIFORNIA
Las Vegas
Los Angeles
San Diego
ARIZONA
Phoenix
Tucson
NEW MEXICO
OKLAHOMA
Dallas
TEXAS
Austin
Houston
LOUISIANA
MISSISSIPPI
ALABAMA
GEORGIA
Atlanta
TENNESSEE
Nashville
WEST VIRGINIA
VIRGINIA
NORTH CAROLINA
Charlotte
SOUTH CAROLINA
FLORIDA
Orlando
Tampa
Miami
BAJA CALIFORNIA
SONORA
CHIHUAHUA
COAHUILA
NUEVO LEON
Monterrey
SINALOA
DURANGO

SOLUTION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MICHIGAN
Toronto
Ottawa
Montreal
Québec City
MAINE
NEW HAMPSHIRE
NEW YORK
MASSACHUSETTS
Chicago
NEBRASKA
IOWA
ILLINOIS
INDIANA
OHIO
PENNSYLVANIA
NEVADA
UTAH
Denver
United States
COLORADO
KANSAS
Kansas City
Indianapolis
MISSOURI
MARYLAND
WEST VIRGINIA
VIRGINIA
Washington
Sacramento
San Francisco
San Jose
CALIFORNIA
Las Vegas
KENTUCKY
Nashville
OKLAHOMA
TENNESSEE
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San Diego
Tucson
El Paso
Dallas
ALABAMA
GEORGIA
TEXAS
Austin
San Antonio
Houston
LOUISIANA
New Orleans
Jacksonville
BAJA CALIFORNIA
SONORA
CHIHUAHUA
COAHUILA
Orlando
Tampa
FLORIDA
NUEVO LEON
Monterrey
Miami
Gulf of California
SINALOA
DURANGO
Gulf of Mexico

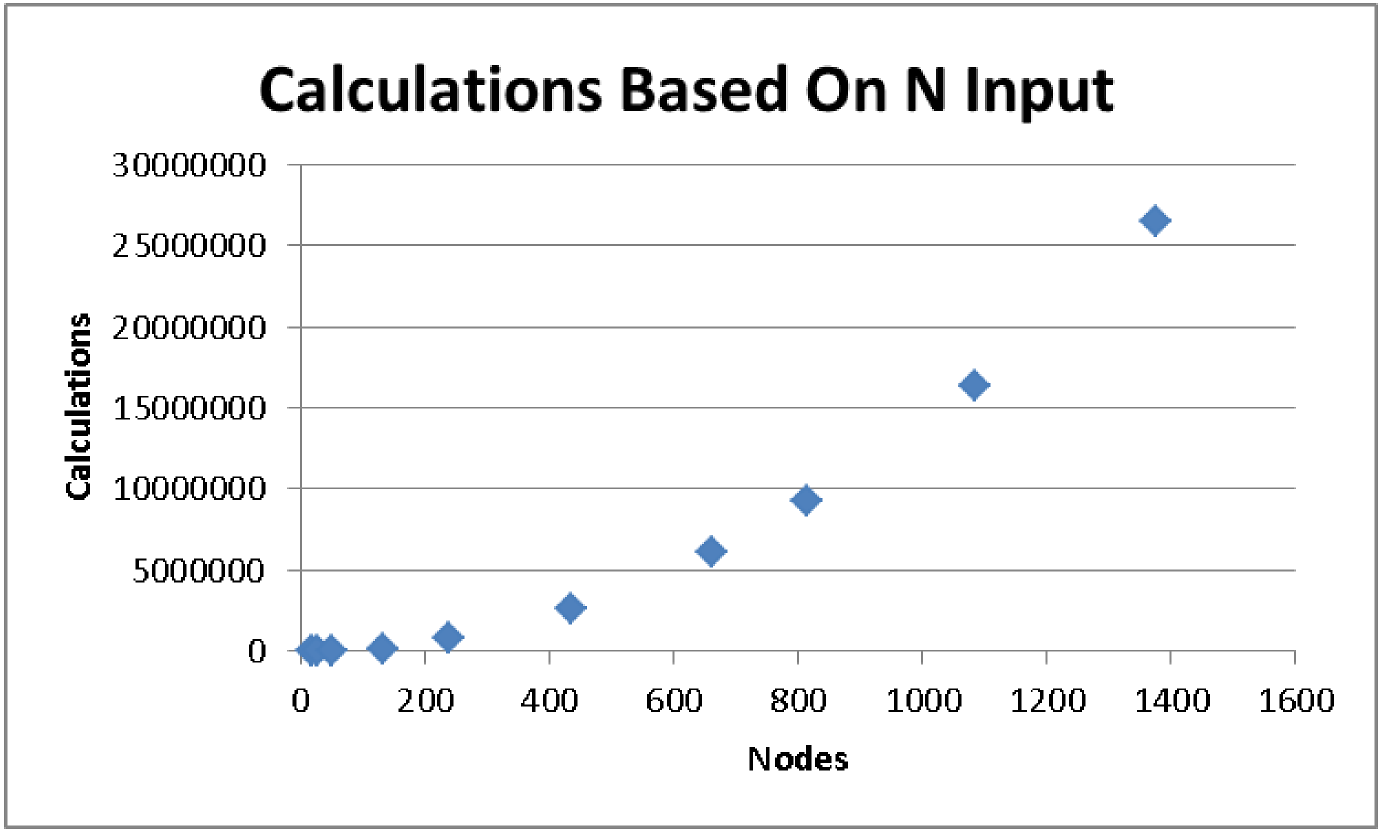
OCP SOLUTION

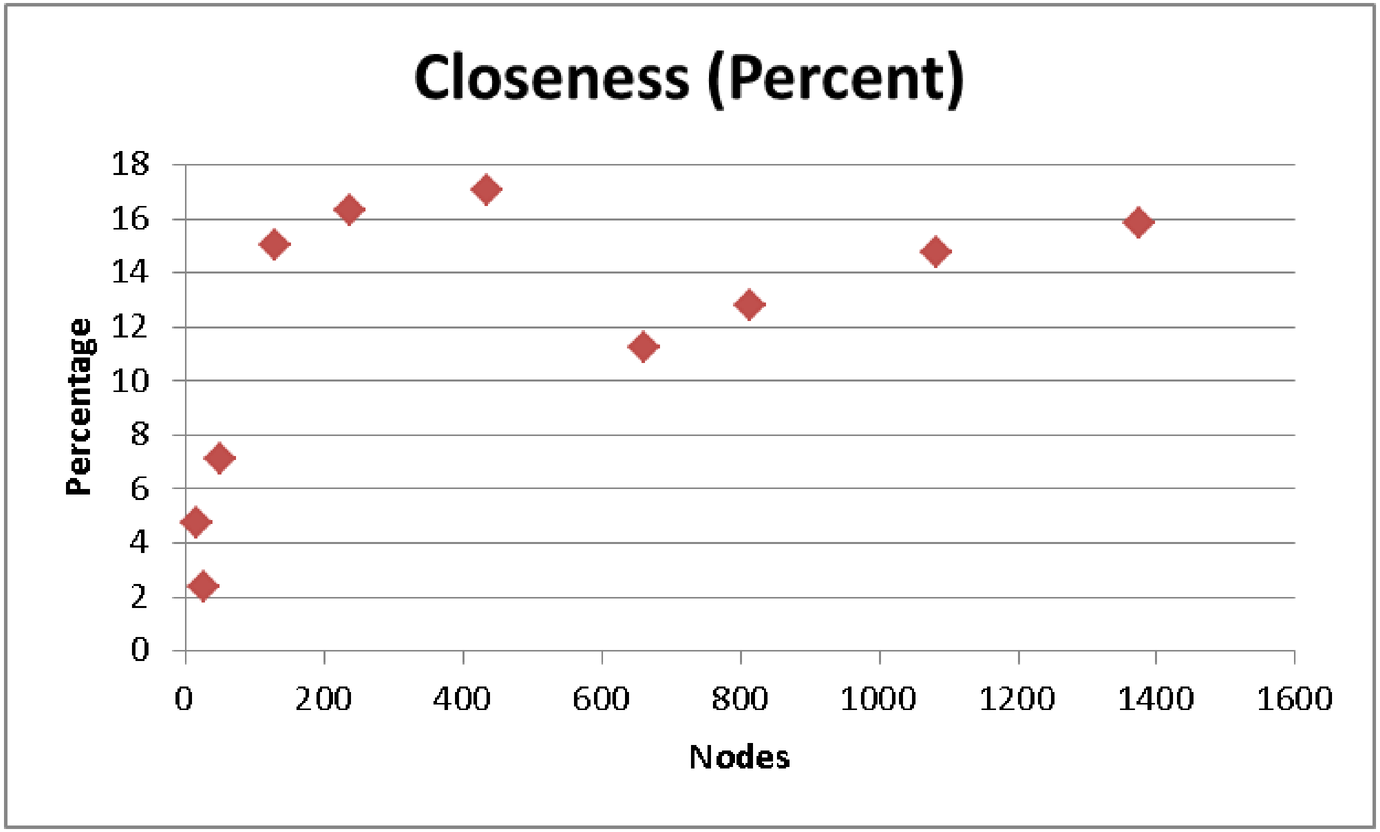
OCP ALGORITHM 50USAL STATS

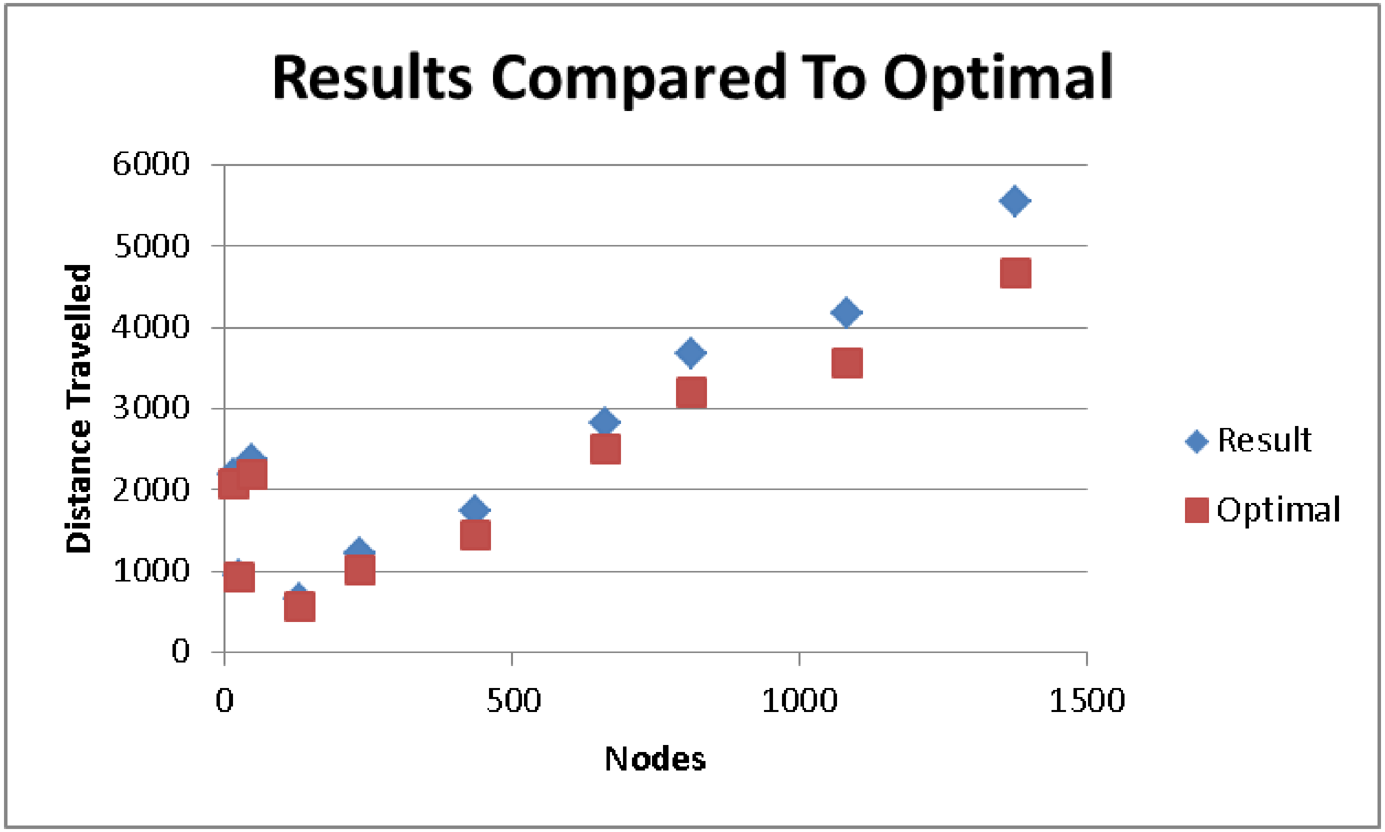
- ~24500 Calculations Done By OCPA
- OCP Solution Path 23696516 Miles
- Real Solution Path 22015038 Miles
- OCP within 7% of Real Solution!

OTHER DATA SETS

ID	Nodes	Calculations	Result	Optimal	How Close (Percent)
GR17	17	2176	2189	2085	4.751027867
FRI26	26	9100	960	937	2.395833333
XQF131	131	136240	664	564	15.06024096
XQG237	237	783048	1218	1019	16.33825944
PBM436	436	2655240	1740	1443	17.06896552
XQL662	662	6126148	2832	2513	11.26412429
DKG813	813	9242184	3669	3199	12.81002998
XIT1083	1083	16405284	4174	3558	14.75802587
DKA1376	1376	26488000	5544	4666	15.83694084


Calculations Based On N Input
30000000
25000000
20000000
15000000
10000000
5000000
0
Calculations
0
200
400
600
800
1000
1200
1400
1600
Nodes


Closeness (Percent)
Percentage
Nodes
0
2
4
6
8
10
12
14
16
18
0
200
400
600
800
1000
1200
1400
1600


Results Compared To Optimal
Distance Travelled
6000
5000
4000
3000
2000
1000
0
0
500
1000
1500
Nodes
Result
Optimal

RESULTS

- **OCP Algorithm Produces Very Efficient Solutions**
- **OCP Algorithm Takes Very Little Calculations**
- **OCP Algorithm is Competitive Against Other Algorithms**

WORKS CITED

http://www.math.uwaterloo.ca/tsp

https://people.sc.fsu.edu/~jburkardt/datasets/tsp/

https://www.iwr.uni-heidelberg.de/groups/comopt/software/TSPLIB95/